4NAND – Augmenting Logic with Imaginary Truth Values

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For G. Spencer Brown, who saw it first.

Consider the following table for an operator (we'll call it 4NAND):

⤊	⊥	⊤	⊣	⊢
⊥	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊣	⊢
⊣	⊤	⊣	⊣	⊤
⊢	⊤	⊢	⊤	⊢

Note that the 4NAND operator ⤊ matches the binary NAND operator ↑ on the set $\{\bot, \top\}$:

$$\bot \uparrow \bot = \bot \uparrow \top = \top \uparrow \bot = \top, \quad \top \uparrow \top = \bot.$$

Here are the tables for 4NOT, 4AND, and 4OR.

X	$\neg X$
⊥	⊤
⊤	⊥
⊣	⊣
⊢	⊢

⩓	⊥	⊤	⊣	⊢
⊥	⊥	⊥	⊥	⊥
⊤	⊥	⊤	⊣	⊢
⊣	⊥	⊣	⊣	⊥
⊢	⊥	⊢	⊥	⊢

⩔	⊥	⊤	⊣	⊢
⊥	⊥	⊤	⊣	⊢
⊤	⊤	⊤	⊤	⊤
⊣	⊣	⊤	⊣	⊥
⊢	⊢	⊤	⊥	⊢

So ⤊ functions as a "real" NAND since the induced negation inverts ⊤ and ⊥ but ignores ⊣ and ⊢.

There is also an "imaginary" NAND whose induced negation inverts ⊣ and ⊢ but ignores ⊤ and ⊥.

Denote 4iNAND by ⥮, with ∼ as 4iNOT, ⊓ as imaginary 4iAND, and ⊔ as imaginary 4iOR.

We make a fairly arbitrary choice of "orientation" by reasoning that ⊤ must be to ⊣ as ⊥ is to ⊢.

⥮	⊥	⊤	⊣	⊢
⊥	⊥	⊣	⊥	⊣
⊤	⊣	⊤	⊤	⊣
⊣	⊥	⊤	⊢	⊣
⊢	⊣	⊣	⊣	⊣

Here are the tables for 4iNOT, 4iAND, and 4iOR.

X	$\sim X$
⊥	⊥
⊤	⊤
⊣	⊢
⊢	⊣

⊓	⊥	⊤	⊣	⊢
⊥	⊥	⊢	⊥	⊢
⊤	⊢	⊤	⊤	⊢
⊣	⊥	⊤	⊣	⊢
⊢	⊢	⊢	⊢	⊢

⊔	⊥	⊤	⊣	⊢
⊥	⊥	⊣	⊣	⊥
⊤	⊣	⊤	⊣	⊤
⊣	⊣	⊣	⊣	⊣
⊢	⊥	⊤	⊣	⊢